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Statistical Forecasting of High-Way Traffic Jam at a Bottleneck

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Abstract

Maintenance works on high-ways usually require installation of bottlenecks that disturb traffic. The article presents a new mathematical model for analysis and forecasting of traffic jam evolution in front of a bottleneck. The model is comprised of two partial differential equations for the mean velocity and density of cars. The first equation describes relaxation of velocity to its equilibrium value determined by a new fundamental diagram. The second is the continuity equation and describes adaptation of the density to the input traffic flow that is forecast statistically. Numerical treatment of the model yields distributions of traffic variables that exhibit characteristic properties of jam evolution. The performance of the method is demonstrated by forecasting the jam that would develop during rush-hour if a bottleneck were installed on a high-way close to Ljubljana. Beside the model a new method is presented for approximate prediction of jam length based upon input flow and bottleneck capacity that is specified by the fundamental diagram. The corresponding computer programs represent a new tool by which experts can analyze properties of bottlenecks in order to optimize them.

1 Introduction

In the past year we have introduced a new approach to non-parametric statistical modeling and forecasting of traffic flow on the network of high-ways in Slovenia (Grabec and Švegl, 2010; Grabec et al., 2010a). Based upon the corresponding predictor we have developed an intelligent graphic user interface for forecasting of traffic flow rate that is now applied by our traffic information center at Ljubljana (Grabec et al., 2010b). The goal of this article is to stretch applicability of this interface to forecasting of traffic congestions and jams caused by bottlenecks and other disturbances on high-ways. For this purpose forecast data about traffic flow rate have to be properly transformed into other traffic characteristics. Here we consider forecasting of the length of the traffic jam that

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develops at the bottleneck due to reduced flow through it. With this aim we first present a new formulation of the fundamental diagram of traffic flow which provides for modeling of traffic dynamics at a bottleneck based upon forecast input flow. The applicability of the proposed model is demonstrated by an example of the bottleneck whose properties are characterized by a decreased allowed speed limit.

2 Theoretical background

Road traffic can be most thoroughly described by a set of dynamic rules that determine trajectories of particular cars (Helbing, 1997; Kerner, 2001; Treiber and Kesting, 2010). However, such micro-dynamical description is usually too complex for on-line applications in traffic information centers where more general properties of traffic are mainly sought. Consequently, we turn to a macroscopic description based upon just two variables that describe the mean density ρ and velocity v of cars (Kerner, 2001; Treiber and Kesting, 2010). The corresponding mean flow rate $Q=\rho v$ is here considered as the basic variable for the description and analysis of the traffic state at the bottleneck.

In a simple case of a steady and homogeneous traffic state the variables ρ and v do not depend on position and time, but they are mutually related. The graph of the corresponding relation $v=v(\rho)$ represents the first fundamental diagram of traffic (Helbing, 1997; Kerner, 2001; Treiber and Kesting, 2010; Siebel and Mauser, 2005). By the expression $Q=\rho v(\rho)$ this diagram is transformed into the second diagram that represents the flow rate $Q(\rho)$ as a function of the density

There are generally three approaches used when specifying the fundamental diagram: empirical, micro-dynamic, and analytical one. In the first case some proper function is statistically adapted to measured data by an optimal selection of its free parameters. In the second case the basic relation $v = v(\rho)$ is extracted from dynamic rules describing trajectories of particular cars, while in the last case this relation is derived based upon some general assumptions about the traffic properties. For our purpose we next follow this case and first consider quasi static and homogeneous free flow of vehicles on a high-way where the maximal allowed speed is given by the limit value v_o. We assume that the corresponding traffic state is characterized by the density ρ that is determined by the distance between cars r as: $\rho=1/r$. We further assume that the most fundamental property of the traffic stems from the experience of drivers which adjust distance between cars so that it grows with their velocity. This property is most simply described by a linear relation: $r = \lambda + \tau w$. For our purpose it is better expressed by: $w = (r - \lambda) / \tau$. Here λ denotes the mean length of cars, τ the reaction time, and w a characteristic velocity determined by the transition of the clear spacing between cars $r - \lambda$ in the reaction time τ. For the safety reasons, a driver tries to keep the velocity of the car

v bellow the characteristic value w, and also bellow the allowed limit v_o. Consequently, the values w and v_o can be considered as components of a composed constraint. In order to describe it we represent particular constraints by the inverse values $1/v_o$ and $1/w$ and add them to get the following rule for velocity constraint: $1/v = 1/v_o + 1/w$. This rule indicates that the value v cannot surpass neither v_o nor w. However, one could expect that still better expression of composed constraint could be obtained if a proper weight is assigned to particular terms in the last rule. Such a weight should point out relative importance of one term with respect to another one, and consequently, it is enough if just one term is weighted. We arbitrary assign a weight to the last term and assume that its importance grows with the increasing density of cars that corresponds to the decreasing value of w. The weight is then expressed relatively with respect to some characteristic parameter u which should be determined experimentally. Based on this reasoning, the weight is expressed as u/w which yields the rule: $1/v = 1/v_o + u/w^2$. Its more convenient form is given by the expression for the velocity:

$$v = v_o/(1 + uv_o/\, w^2) = v(\rho) \tag{2.1}$$

It is important that the characteristic value w depends on $\rho = 1/r$ and consequently, the last equation describes the fundamental traffic law $v = v(\rho)$ and the first fundamental diagram.

In order to complete our description the parameter u has to be specified. Since its unit must coincide with the unit of velocity, we arbitrary put $u = C\,\lambda/\tau$. Comparison of the rule Eq. (2.1) with the rules obtained from measured data (Helbing, 1997) has revealed that a good agreement is obtained if the proper value of constant is set to $C = 3.1$. Simultaneously with this estimation, the following values: $\lambda = 4.4$m and $\tau = 1.3$s have been estimated as proper ones. Slightly worse model yields the values $\lambda = 5$m and $\tau = 1$s that are more convenient for presentation of results obtained numerically. Figure 1 shows agreement between experimentally and theoretically determined fundamental diagram for the velocity $v(\rho)$ in the case with the limit value v_o=110km/h. Similarly, Figure 2 shows the corresponding second diagram for the flow rate $Q(\rho)$. Experimental data are taken from the reference (Helbing, 1997) and correspond to a high-way with speed limit 120km/h, that is decreased to the value 110km/h due to the presence of trucks.

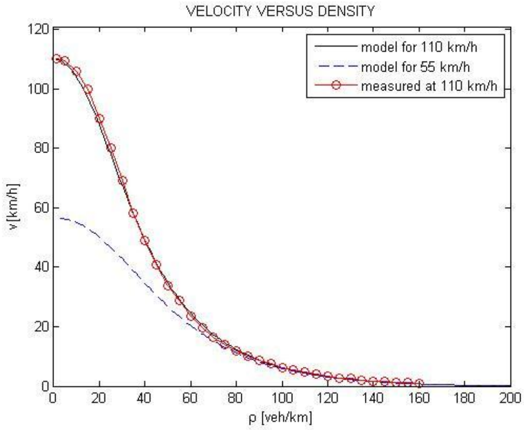


Figure 1: Dependence of the velocity v on the density of cars ρ. Experimental data are taken from the reference (Helbing, 1997).

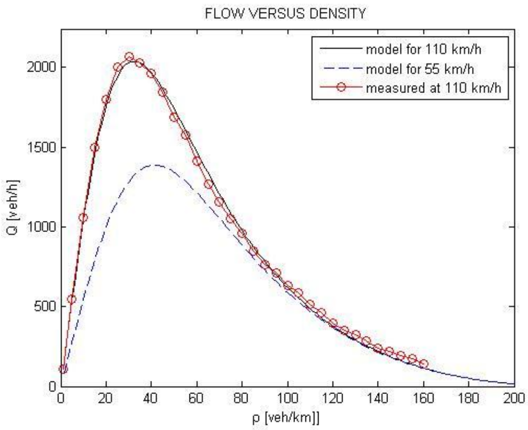


Figure 2: Dependence of the traffic flow rate Q on the density of cars ρ. Experimental data are taken from the reference (Helbing, 1997).

The limit value v_o = 110km/h has been selected to render possible comparison with models of other authors (Helbing, 1997; Treiber and Kesting, 2010). In addition to this, the diagrams corresponding to the limit value v_o = 55km/h are presented (--) in order to indicate the properties of the fundamental diagram corresponding to a typical bottleneck. The most important characteristic of the second diagram is given by the maximal value of the flow rate that determines the *road capacity*. In Slovenia the velocity limit on high-ways is v_o = 130 km/h while most often observed limit in a bottleneck is v_o = 60 km/h. For a single lane the corresponding capacities are $Q_{max}\sim 2.2*10^3$veh/h and $Q_{max}\sim 1.4*10^3$veh/h respectively. One could expect that the jam appears when the flow to a bottleneck reaches its capacity.

The fundamental diagrams determined by our model coincide surprisingly well with the experimentally determined ones. This agreement indicates that reasoning at the formulation of the model, although rather simple, is consistent with characteristic properties of the traffic flow in equilibrium and describes correctly the adaptation of the mean velocity of cars to the traffic density.

In our derivation of the fundamental law Eq. (2.1) we considered quasi steady and homogeneous traffic state. However, this is not the case when treating congestion phenomena and evolution of traffic jams. Since the derived law Eq. (2.1) describes well the mean traffic properties in equilibrium, we further assume that the velocity at a certain position x and time t is adapted to the equilibrium value $v_e(\rho)$ determined by Eq. (2.1) during some characteristic adaptation time T (Treiber and Kesting, 2010; Siebel and Mauser, 2005). We describe this adaptation by the most simple differential equation:

$$dv/dt = (v_e(\rho) - v)/T, \tag{2.2}$$

and further consider the velocity and density as mutually dependent field variables $v = v(x,t)$ and $\rho = \rho(x,t)$. In accordance with this dv/dt in the differential Equation (2) denotes the convective derivative: $\partial v/\partial t + v\partial v/\partial x$. The fundamental dynamic law of the field is then given by the continuity equation (Siebel and Mauser, 2005):

$$\partial\rho/\partial t + \partial(\rho v)/\partial x = I, \tag{2.3}$$

in which $I = I(x,t)$ denotes the traffic source term. If we start analysis at a certain point x_o where the traffic flow rate $Q(t)$ is forecast, then the source term can be described by the expression: $I(x,t) = Q(t)\,\delta(x - x_o)$ in which δ denotes the Dirac's delta function.

The drivers try to adapt their velocity predominantly to the leading car, but with a delay specified by the reaction time τ. Consequently, when describing the adaptation of velocity v at the position x and time t, the density in Eq. (2.2) has to be taken at some position Δx ahead of x and at delayed time $t - \tau$. A typical value of Δx is several lengths of the car: $\Delta x \sim 3\lambda$. Similarly the relaxation time is several reaction times: $T \sim 3\tau$.

The model equations (1-3) represent a non-linear system of partial differential equations that could not be solved analytically, and consequently, we have to apply numerical treatment. For this purpose it is convenient to introduce normalized non-dimensional variables by transitions: $t/\tau \to t$; $x/\lambda \to x$; $v * \tau/\lambda \to v$; $\rho * \lambda \to \rho$; $Q * \tau \to Q$. In this case the position is measured in terms of the car length and the time in terms of the reaction time. The solution of the system (1-3) can be found using standard numerical methods for treatment of partial differential equations. However, for this purpose initial and boundary conditions corresponding to a specific case must be given. A typical example is described in the next section.

3 Development of the traffic jam at a bottleneck

In order to demonstrate the performance of the described model we consider the traffic flow at the high-way station near Ljubljana. Its position is shown by the cross section of vertical and horizontal lines in the top diagram of the forecasting unit window shown in Figure 3. The radius of the circle at a certain point indicates the amount of the flow rate at the selected time of forecasting. Its dependence on time in the selected day is shown by the bottom record. Two expressive peaks in this record denote increased population mobility during rush-hours in the morning and afternoon. Due to this property the recorded function can be simply modeled by superposition of two normal distributions and a constant, as shown in Figure 4. Such analytical model is convenient when characterizing human activity and mobility and was applied also in our further treatment (Grabec et al., 2010a).

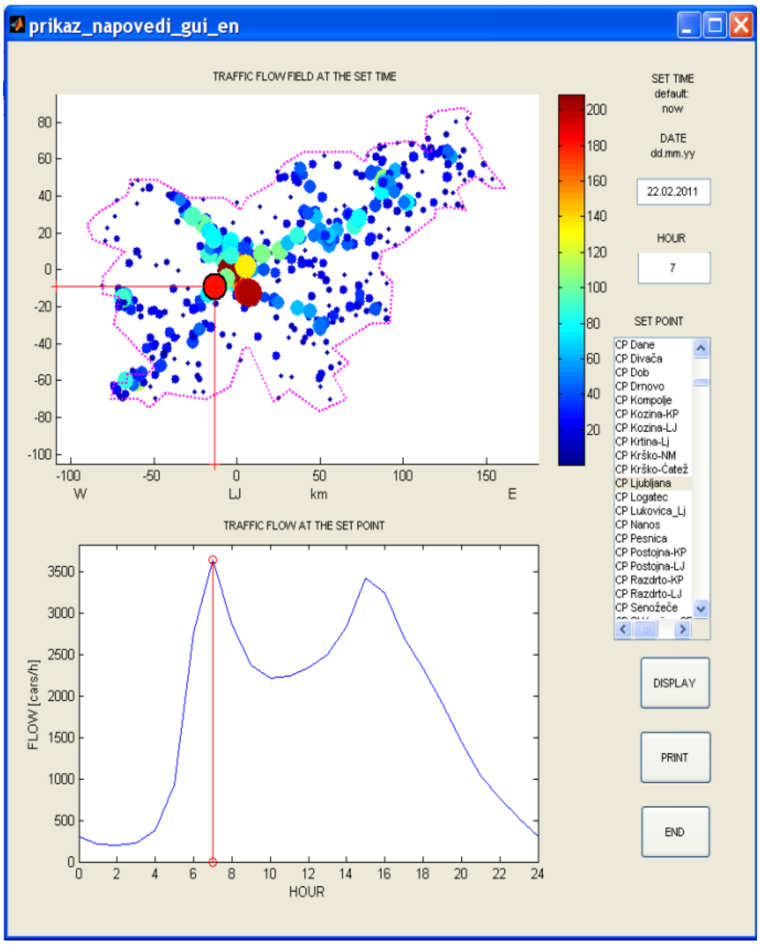


Figure 3: The window of intelligent graphic user interface for traffic flow forecasting. Top graph shows the distribution of predicted traffic flow rate $Q(x)$ in Slovenia at the selected day of February 22, 2011, while the bottom one shows the dependence of $Q(t)$ at the selected position denoted by the red circle (Grabec et al., 2010b).

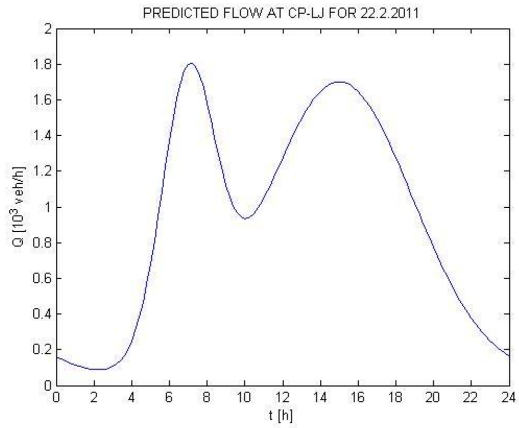


Figure 4: Dependence of the predicted traffic flow rate $Q(t)$ on time.

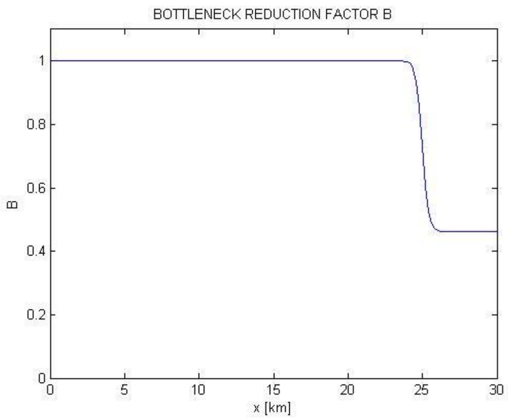
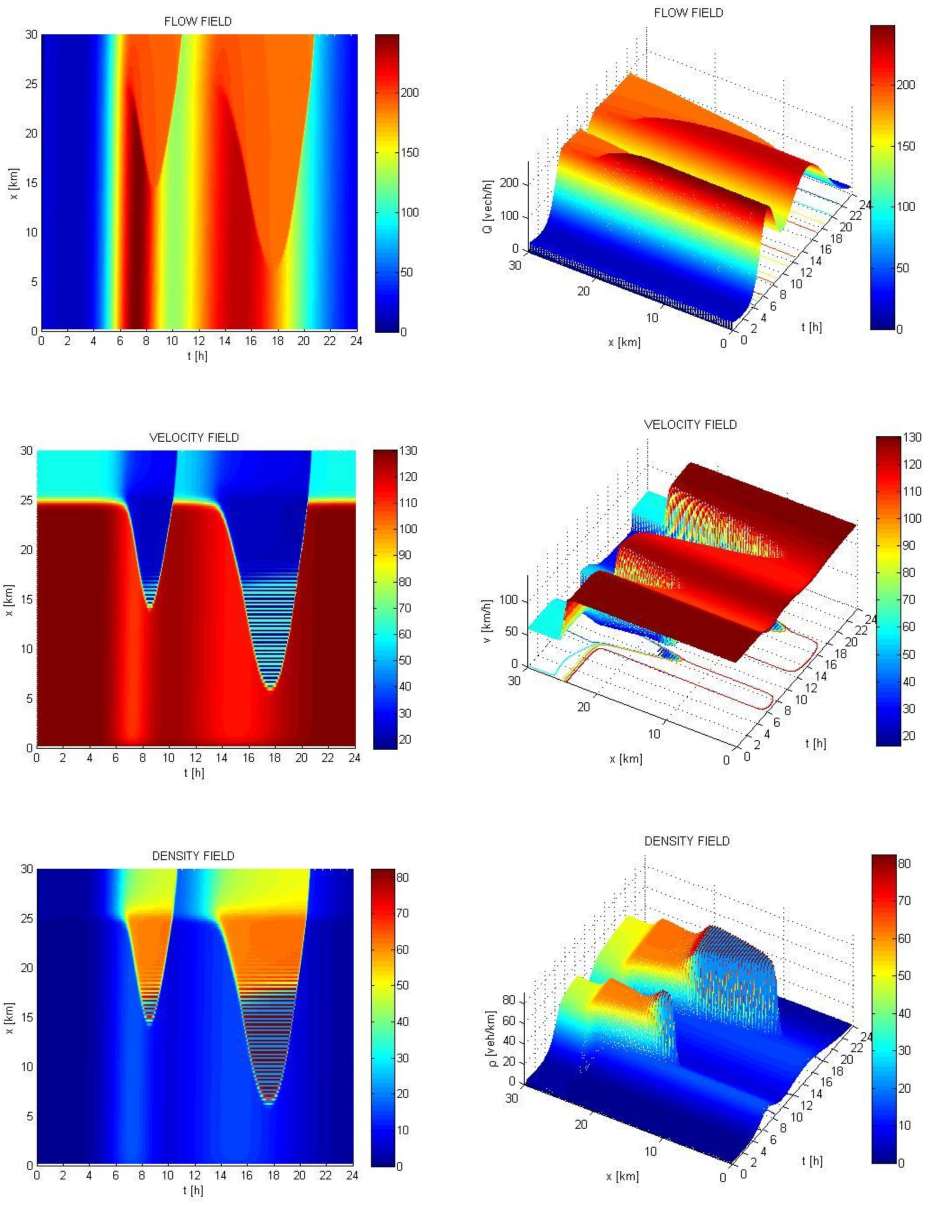


Figure 5: Bottleneck characteristic for the transition from 130 to 60km/h.

Our next goal is to demonstrate what would happen during selected day if a bottleneck is placed at the road section covering the selected point. For this purpose we consider a road section of 30km length with the bottleneck installed at 25km. In the bottleneck the velocity limit is reduced from 130km/h on the free road to 60km/h. The distribution of velocity reduction factor B is shown by the characteristic in Figure 5. To solve the problem, we select the cell size in the spatial direction equal to Δx =200m, and the time step equal to $\Delta t = \tau = 1s$. We next consider homogeneous initial and boundary conditions equal to 0, so that the traffic state is completely determined by the incoming flow rate $Q(t)$ as estimated by the forecasting module and shown in Figure 4. For the calculations we further consider the time interval that contains one arbitrary selected day. The calculated distributions of field variables are shown in Figs. 6a, b, and c using color coding for the amplitude of field variables. Figure 6a shows the distribution of the flow field $Q(x,t)$, while Figs. 6b and 6c show the corresponding distributions of the velocity $v\,(x,t)$ and density $\rho(x,t)$ fields. The flow enters the road section at

$x = 0$ and moves in the x direction. At the rush-hour its amplitude first grows with time t to the maximum and then falls again. When passing through the bottleneck its maximal value is decreased and the peak is flattened. The reduction of velocity in the bottleneck is observable in the graph of its distribution in Figure 6b as blue downward step. At low t the velocity is high at low x, but when cars pass the bottleneck, their velocity, is decreased due to the decreased speed limit. Simultaneously with decreasing velocity the density is increased as shown in Figure 6c.



a

b

c

Figure 6: Distributions of traffic field variables: **(a)** flow rate, **(b)** velocity, density **(c)**. Left – ground plan, right – side view.

With increasing time and flow at rush-hour the reduction of velocity in the bottleneck leads to evolution of jam with an expressive peak. At the peak the jam exhibits wave-like structure that corresponds to stepwise movement of cars. When the rush-hour maximum is passed, the input flow again starts decreasing, which further leads to a decreased density, increased velocity and dilution of the jam by the flow through the bottleneck. A similar evolution of jam as in the morning is observed also in the afternoon rush-hour time. From the graphs of field variables we can forecast the length of jam and its velocity of spreading.

4 Estimation of jam length

Macroscopic modeling of traffic by partial differential equations yields rather general description of the traffic jam evolution at the bottleneck in terms of dynamic field variables. As demonstrated in the previous section the system of Eqs. (1-3) has to be solved for this purpose. However, for practical purposes most often just an approximate value of the traffic jam length is sought, and consequently, there appears a question how to avoid numerical treatment of partial differential equations. For this purpose we next turn to an approximate treatment of the complete problem. With this aim we first define the capacity of road by using the fundamental diagram of flow rate and then apply it in an approximate estimation of the jam length.

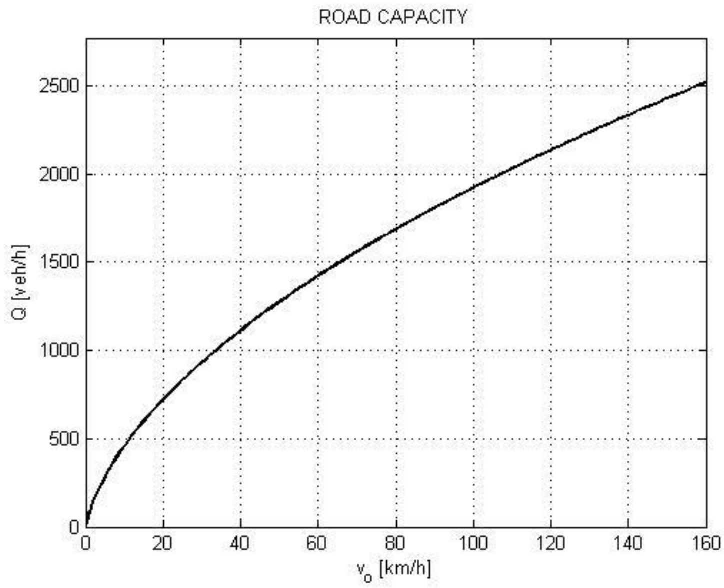


Figure 7: Dependence of road capacity Q_m on the speed limit v_o.

As mentioned previously, the basic property of roads traffic is that its flow rate exhibits a maximum at certain characteristic value of density. The maximal value

Q_m, that represents the road capacity, depends on the speed limit. The corresponding dependence has been determined numerically from the corresponding fundamental law and is shown in Figure 7.

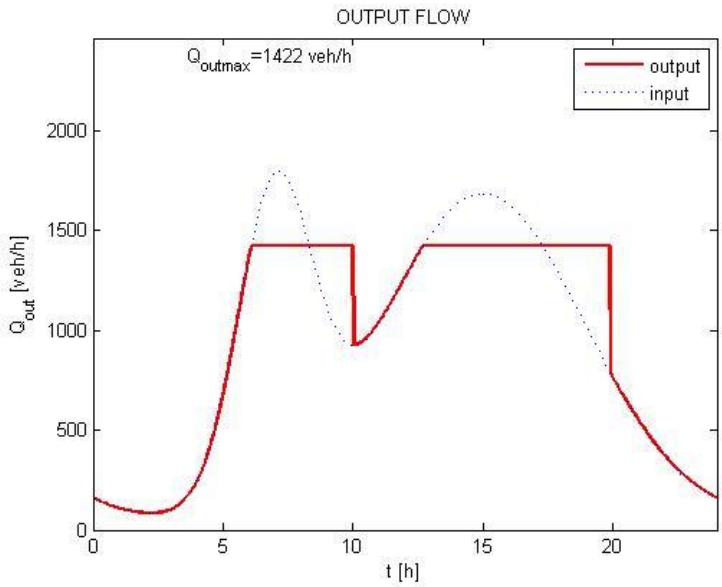


Figure 8: Dependence of output (bold) and input (dotted) flow on time.

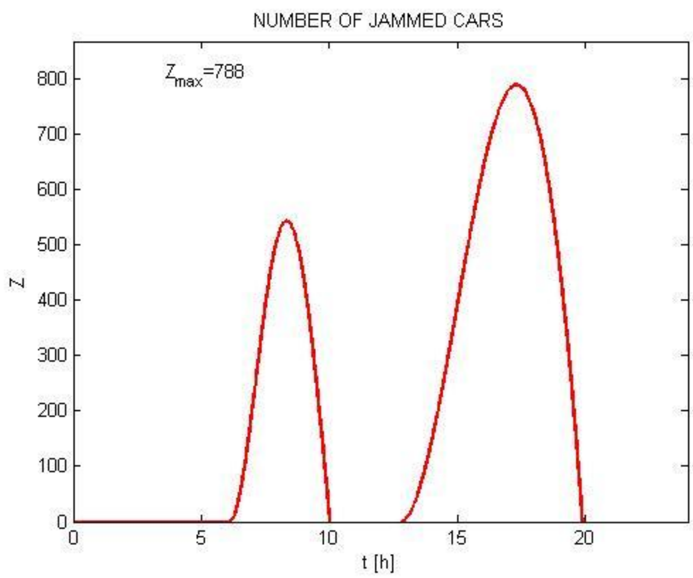


Figure 9: Estimated number of jammed cars Z in dependence of time t.

Quite generally a lower value of speed limit yields a lower value of road capacity. This property leads us to the following simplified reasoning about the jam development at the bottleneck. Let us consider the example when the input flow Q_i is increasing with time. As long as the input flow is below the bottleneck capacity we can assume that all cars pass it fluently. But, when the input flow surpasses the capacity, the bottleneck stops a portion of input flow: $dQ = Q_i - Q_b$. This difference then causes increasing number of cars in front of the bottleneck and evolution of jam. If we know the dependence of input flow on time, we can estimate the number of stopped cars Z by integrating $dQ(t)$ with respect to time. We can then estimate the corresponding jam length L by multiplying the number of stopped cars by the distance between cars that corresponds to the speed limit of the bottleneck.

To demonstrate the proposed approximate characterization we again consider the example from the previous section. Figure 8 shows the input (dotted) and the output (solid) flow. The latter is determined by the bottleneck capacity that determines the level of the horizontal section in the graph. The corresponding estimate of the number of cars Z in the jam, as determined by the integral of $dQ(t)$, is shown in Figure 9.

Diagram on Figure 8 renders possible a rough estimation of the speed of jam propagation in backward direction that could be compared with the result shown on Figure 9. The bottleneck during the rush-hour time does not permit all incoming cars to pass it when the input flow surpasses its capacity 1422veh/h. The difference between input and output flow contributes to the formation of jam. If we assume approximately linear increasing and decreasing of flow from ~1400 to ~1800 and back to ~1400veh/h in the time interval from 6-8h, then we obtain that about ~200veh/h is stopped in this interval which yields in 2h about Z~400 vehicles. If we assume approximately quadratic dependence of flow on time during rush-hour we obtain the value Z~540, which coincides well with the height of the first peak in Figure 9. If all cars were not moving and closely packed, the corresponding length of jam would be L~λZ~3km. But the cars are mowing in the jam approximately with the speed determined by its limit in the bottleneck, and consequently the approximate distance between them is $r = \lambda + \tau w {\sim} 20$m that yields four times longer jam length L~12km. This value coincides well with the length of first peak determined by solving differential equations and shown in Figure 6.

Estimation of the jam length is less reliable than the corresponding number of cars, since the distance between cars is changing with their velocity that depends also on properties of the jam. However, the determination of the number of stopped cars is also only approximate since the jam can also influence the dynamics of the flow in the bottleneck itself. More accurate determination of the jam length can thus be obtained just by a strict accounting of the flow dynamics, as described in the previous section. Irrespectively of this deficiency, we can introduce a jam characteristic by the integral of stopped flow that is applicable for an approximate prediction of jamming phenomenon. An advantage is that for such characterization we need just the predicted input flow in dependence on time and the bottleneck capacity.

5 Conclusions

We have shown that in spite of rather complex, non-linear, and stochastic character of traffic, it is possible to model the equilibrium properties of the complete phenomenon by the fundamental diagram (Treiber and Kesting, 2010). This diagram provides for definition of road capacity and rather simple estimation of properties of traffic jam evolving at the bottleneck. Inclusion of the relaxation equation and the continuum equation into description further stretches the applicability of our macroscopic description to non-equilibrium states. These two equations describe the adaptation of velocity to its equilibrium value and the adaptation of the density to the velocity and with it related traffic flow rate. The flow rate is applicable also for description of the traffic source term in the continuity equation which further renders possible forecasting of traffic jam evolution caused by various disturbances, as is for example a bottleneck. For this purpose the flow rate predicted by previously developed intelligent unit is readily applicable. The presented characteristic example of jam formation reveals most characteristic features of this phenomenon. The method presented here has been just developed, and consequently it still needs a thorough experimental verification of performance in real situations before it could be practically applied.

Acknowledgment

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