Special Topics on SNA:
Signed Networks
Temporal Networks
Blockmodeling

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Postgraduate study - 3rd cycle

Signed networks

Signed graph is an ordered pair (G, σ), where:

- $G = (V, L)$ is a graph with a set of vertices V and a set of lines L
- $\sigma : L \to \{p, n\}$ is a ***sign function***, the lines with the sign p are positive, the lines with the sign n are negative. Positive lines are drawn with solid, negative with dotted lines.

The question is:
Is it possible to partition vertices of a signed graph, so that every line that connects vertices that belong to the same cluster is positive and every line that connects two vertices that belong to different clusters is negative?

If it is possible to partition vertices in this way, we call the signed graph *partitionable* or *clusterable*. Especially important are signed graphs where vertices can be partitioned into 2 clusters. Such graphs are called *balanced*.

In the case of people who are friends and enemies a balanced signed graph means that there exist two clusters of people, such that there are only friends inside the clusters and nobody has a friend in the other cluster. This situation is very stable – it cannot happen that there exist a person and two other friends, and the person is a friend of one of them and enemy of the other.

In most real life examples signed graph is not partitionable. In such cases we would like to find partition, which is as close to ideal partition as possible – which has the lowest number of errors. The error of given partition is:

- every negative line among vertices in the same cluster and
- every positive line among vertices in different clusters.

Example: Sampson monastery

Sampson studied relations among 18 monks in the New England monastery. He measured several relations:

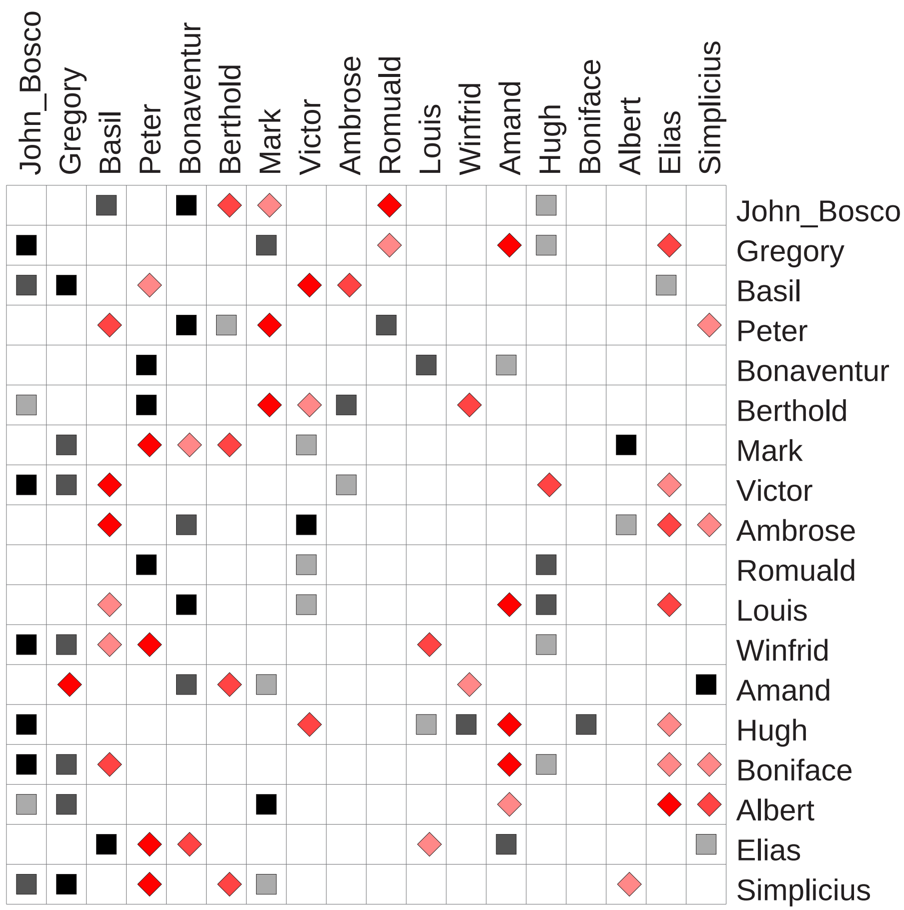
- friendship (affect)
- esteem
- influence
- sanction

Friendship relation was measured in three time points T_2, T_3 and T_4, all others only in T_4.

Relations among monks can be represented as valued signed graphs. Each monk selected 3 other monks whom he liked / disliked the most. A citation of +3 goes to the most liked monk, a citation -3 goes to the most disliked monk.

Friendship relation in T_2:

(black boxes for positive values; red diamonds for negative values):



Results

In the table the total error score for friendship relation in all three time points is given.

No. of clusters	T_2	T_3	T_4
1	48.5	48.0	47.0
2	21.5	16.0	12.5
3	17.5	11.0	10.5
4	19.0	13.5	12.5
5	20.5	16.0	15.0

Two important facts:

- For any number of clusters the error in T_3 is lower than in T_2 and error in T_4 is lower than in T_3. Conclusion: balance is improving during the time.
- In all time points the lowest error occurs for 3 clusters. Therefore

partition into 3 clusters in the most ’natural’.

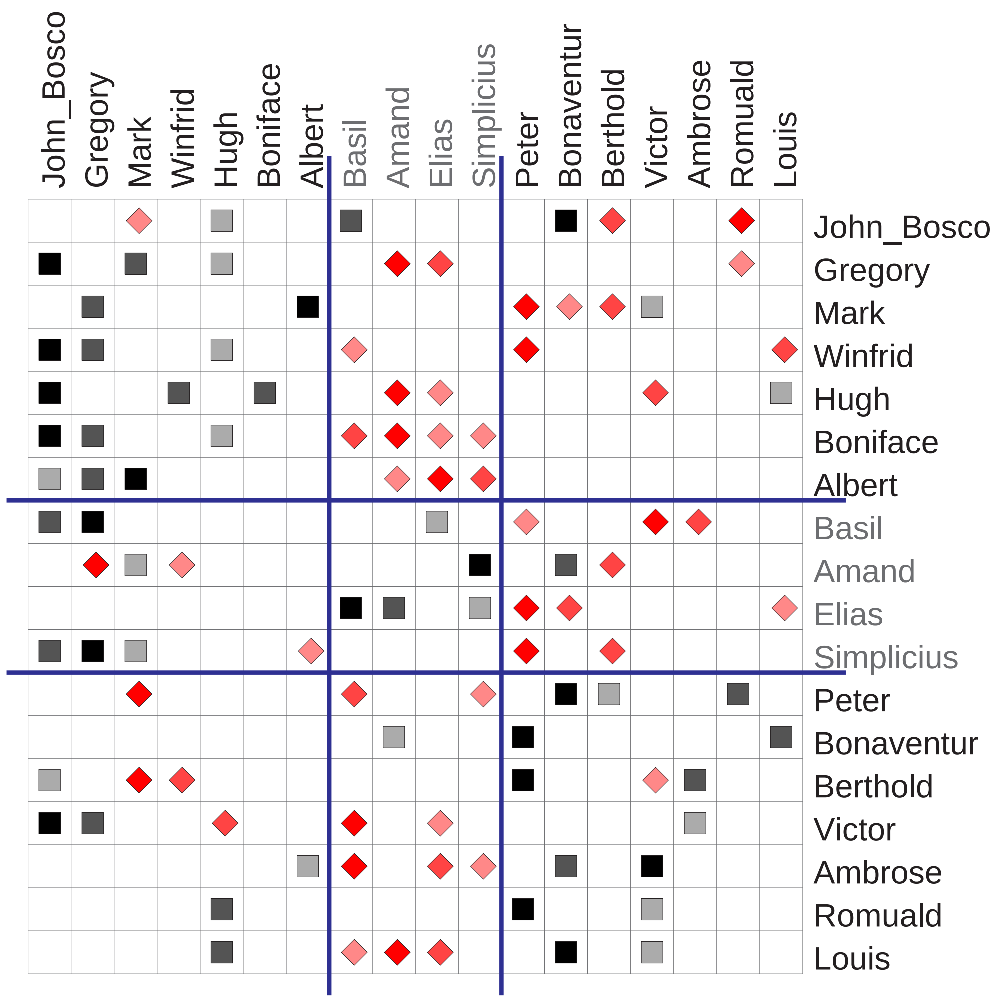
Look at the partition into 3 clusters:

- Optimal partition is equal for all three time points.
- Partition is equal to the partition reported by Sampson, which he obtained using examining the monks.

Maybe this result is a little surprising. It means that the clusters are not changing but relations among monks are more and more ’clear’:

- more lines inside clusters are positive, more lines among clusters are negative;
- negative lines inside and positive lines between clusters are weaker.

Permuted matrix in T_2:

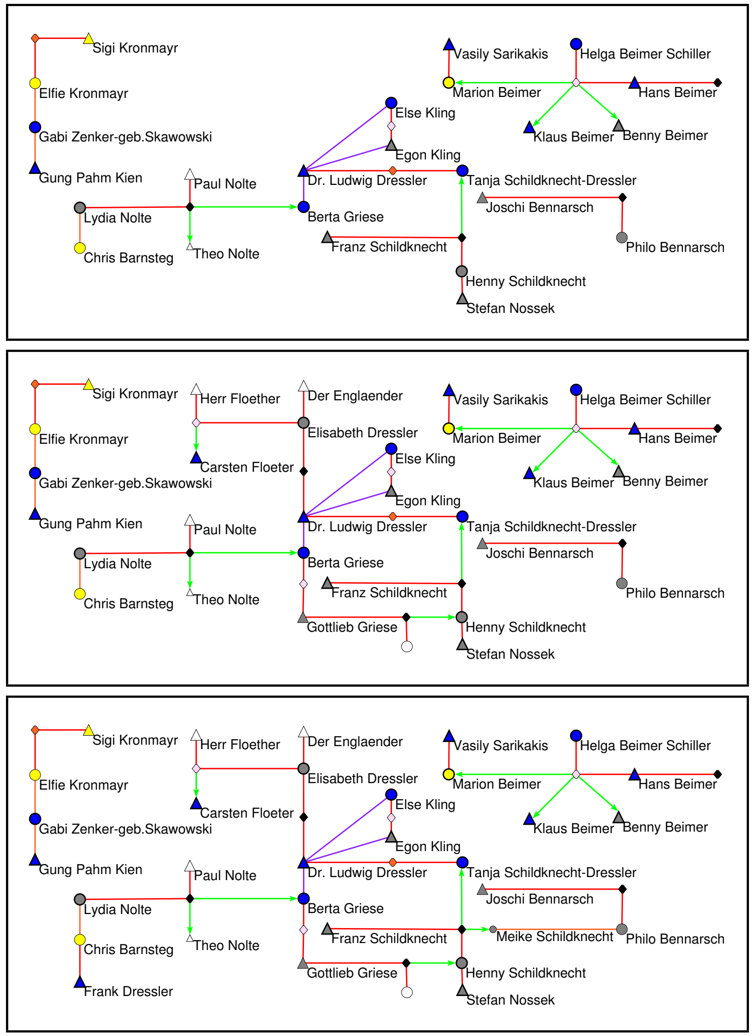


Temporal networks

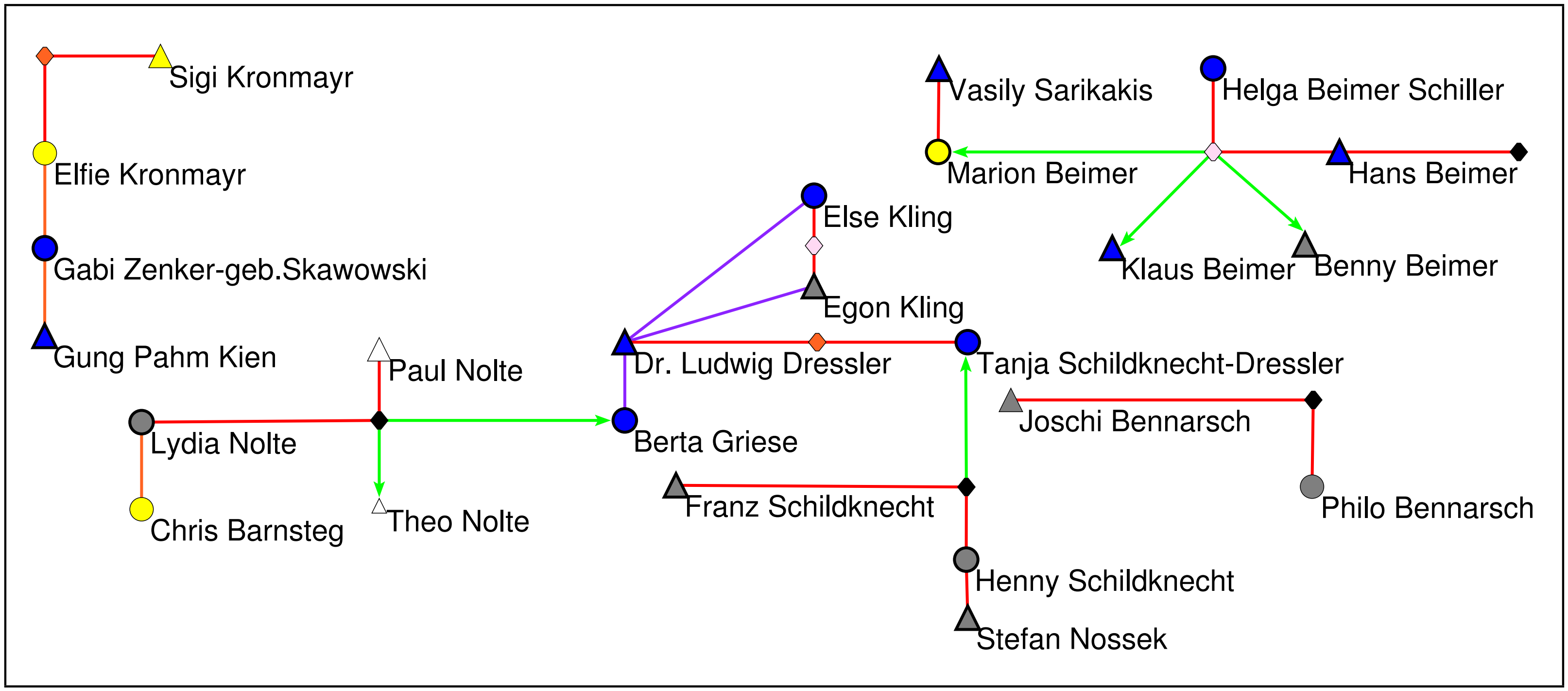
Networks that are changing over time:

- network of friendship in the class in school over several years;
- changes in signed graphs over time
  (Sampson monastery data, Newcomb fraternity);
- network of phone calls inside selected set of phone numbers (used by the police in the investigation of organized crime);
- citation networks from a selected scientific area;
- network of transitions of a ball in a football game;
- changes in HIV networks;
- relations among actors in different episodes of movies;
- births, marriages and deaths in genealogies…

Relations among actors in the long-running German soap opera 'Lindenstrasse' in episodes 5, 6 and 7



Episode 5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Paul Nolte
Lydia Nolte
Chris Barnsteg
Theo Nolte
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Dr. Ludwig Dressler
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Franz Schildknecht
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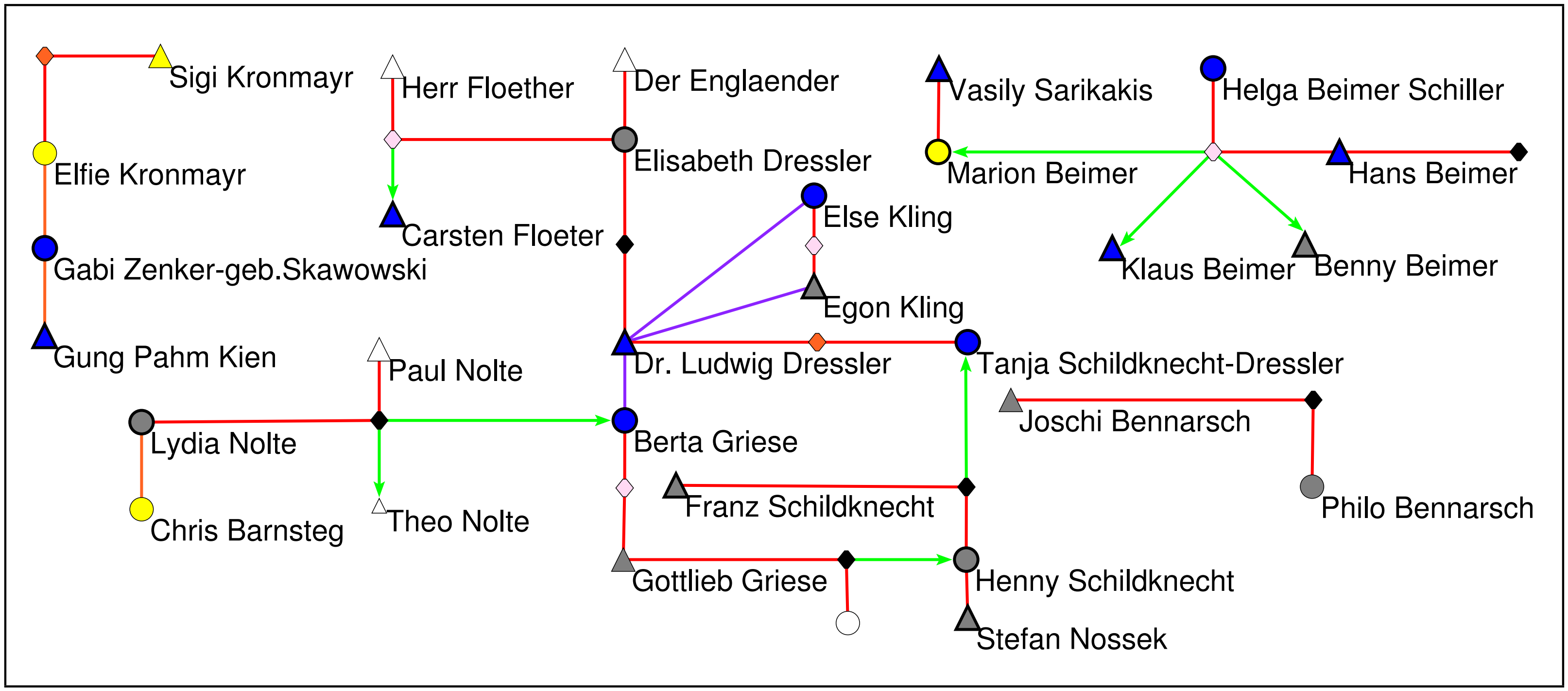
Joschi Bennarsch
Philo Bennarsch
Henny Schildknecht
Stefan Nossek
Vasily Sarikakis
Helga Beimer Schiller
Marion Beimer
Hans Beimer
Klaus Beimer
Benny Beimer

Episode 6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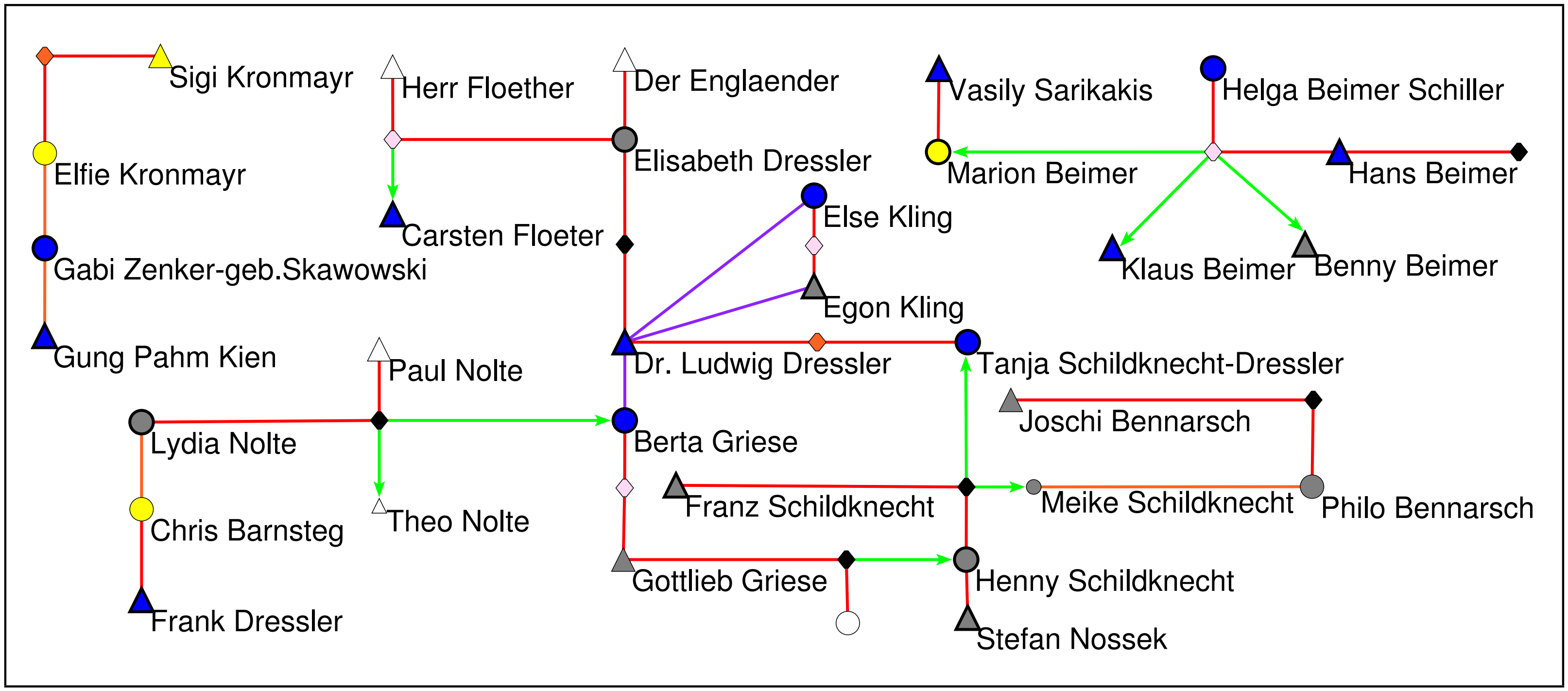
Episode 7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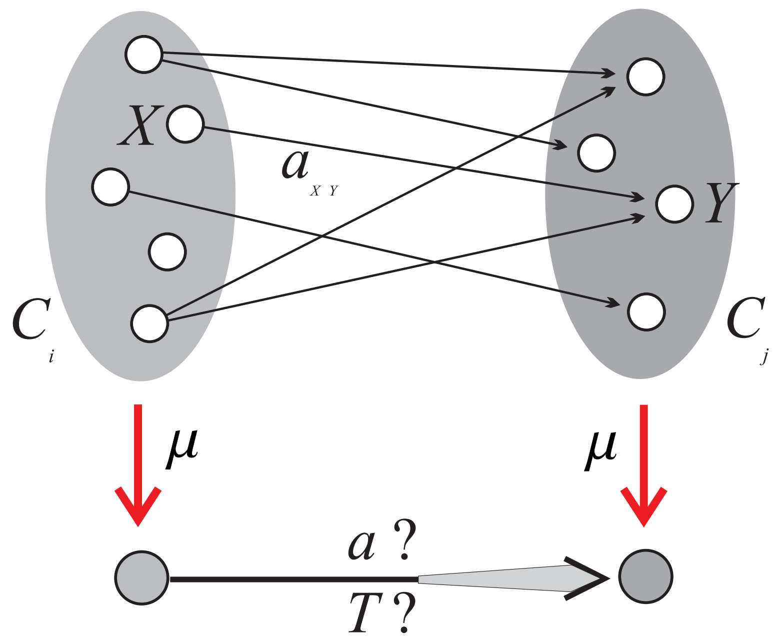
Blockmodeling

The goal of blockmodeling is to reduce a large, potentially incoherent network to a smaller comprehensible structure that can be interpreted more readily. Blockmodeling, as an empirical procedure, is based on the idea that units in a network can be grouped according to the extent to which they are equivalent, according to some meaningful definition of equivalence.



Structural equivalence

Units X and Y are structurally equivalent if they are connected to the rest of the network in identical ways (Lorrain and White, 1971). If X and Y are structurally equivalent they are interchangeable.

X and Y are structurally equivalent iff:

s1. $XRY \Leftrightarrow YRX$

s2. $XRX \Leftrightarrow YRY$

s3. $\forall Z \in E \setminus \{X, Y\} : (XRZ \Leftrightarrow YRZ)$

s4. $\forall Z \in E \setminus \{X, Y\} : (ZRX \Leftrightarrow ZRY)$

Ideal blocks compatible with structural equivalence are:

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{matrix}$$

Regular equivalence

X and Y are regularly equivalent if they link in equivalent ways to other units that are also equivalent.

X and Y are regularly equivalent iff:

R1. $XRZ \Rightarrow \exists W \in E : (YRW \wedge W \approx Z)$

R2. $ZRX \Rightarrow \exists W \in E : (WRY \wedge W \approx Z)$

Ideal blocks compatible with regular equivalence are:

- null block
- blocks that have the property that there is at least one 1 in each of their rows and in each of its columns in regularni bloki

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

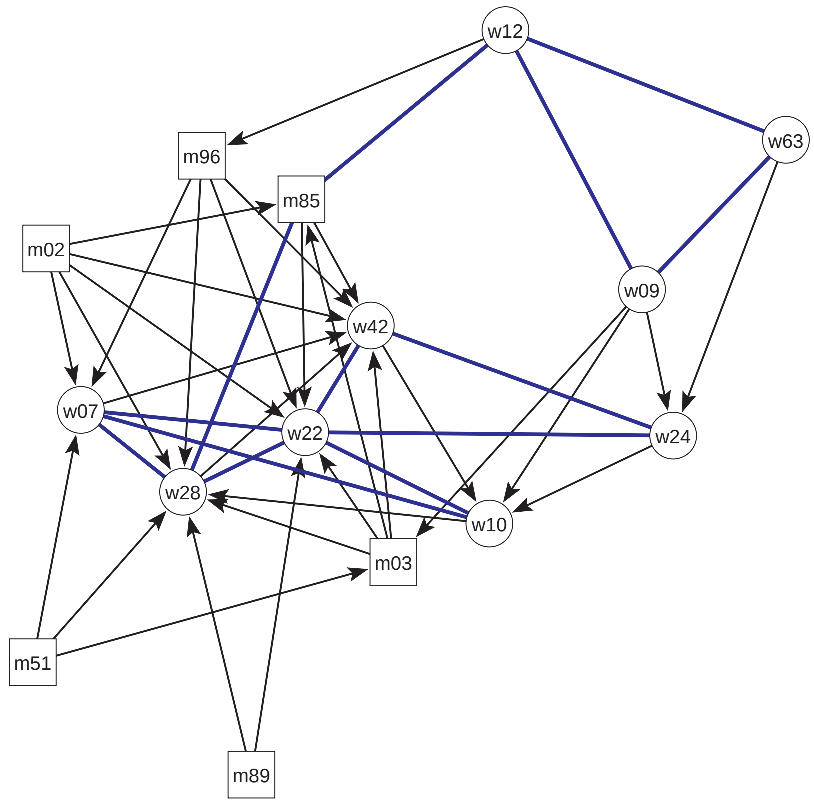
1	0	1	0	0
0	0	1	0	1
0	1	0	0	0
1	0	1	1	0

Structural euqivalence is also regular equivalence.

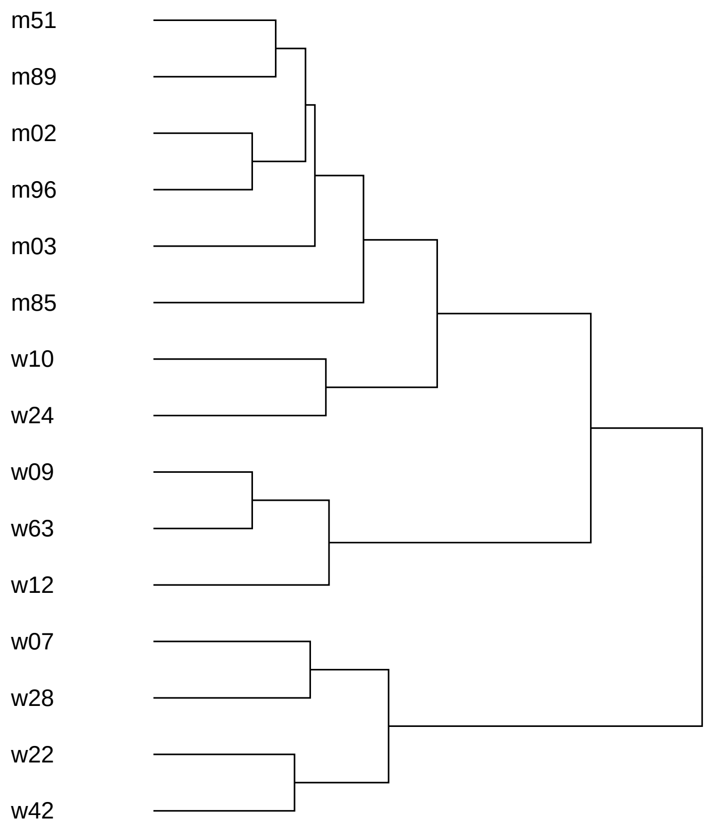
Example

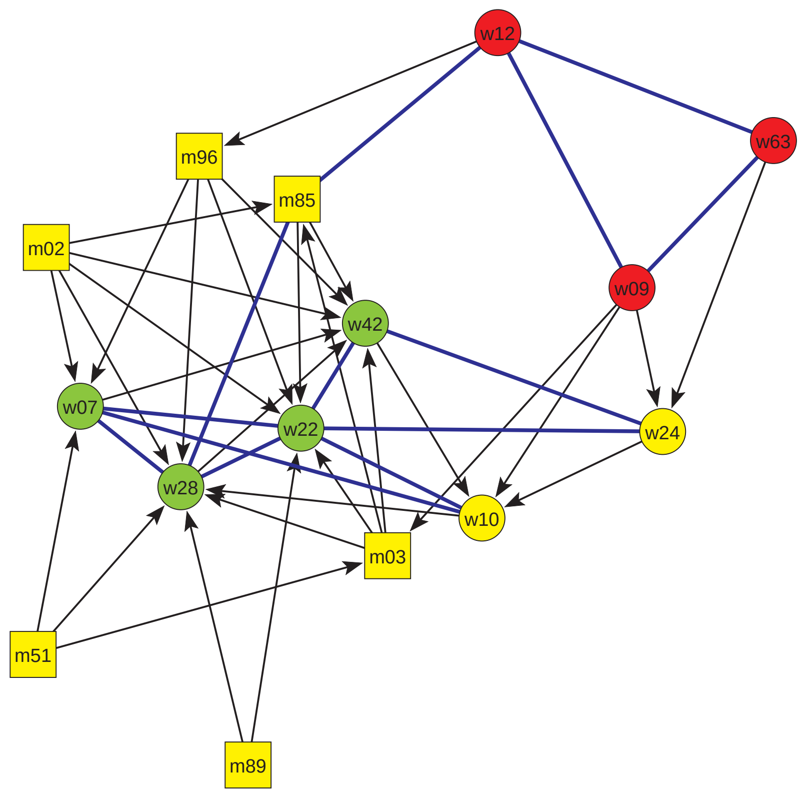
The analyzed network consists of social support exchange relation among fifteen students of the Social Science Informatics fourth year class (2002/2003) at the Faculty of Social Sciences, University of Ljubljana. Interviews were conducted in October 2002.

Support relation among students was identified by the following question: Introduction: You have done several exams since you are in the second class now. Students usually borrow studying material from their colleagues. Enumerate (list) the names of your colleagues that you have most often borrowed studying material from. (The number of listed persons is not limited.)

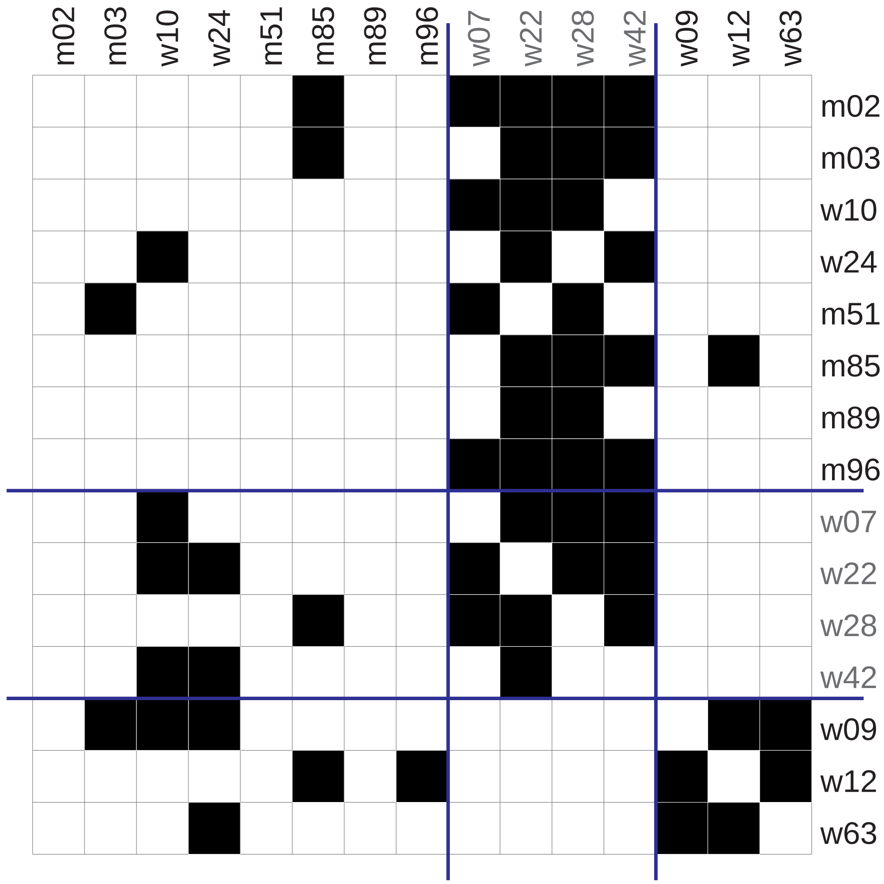

w12
m96
w63
m85
m02
w09
w42
w07
w22
w24
w28
w10
m03
m51
m89

Indirect approach - using dissimilarity matrix





Reoredered matrix



Direct approach - optimization

Does center-periphery model fit our data?

	C_1	C_2
C_1	com, reg	-
C_2	com, reg	-