Photo: V. Batagelj

Further Thoughts on Structural Balance and Signed Networks

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Outline

Measuring Amounts of Imbalance

- Features cycles for measures based on counts of them:
  - Uses counts of balanced and imbalanced cycles.
  - Ratio of imbalanced cycles to all cycles (proportion imbalanced).
  - It is possible to modify this ratio to downweight longer cycles.
  - BUT Counting cycles, in general, is not an easy computational task.
  - Also, it seems hard, cognitively, for people to keep track of cycles.
- Use lines to create *line indexes*
  - Count the smallest number of lines who sign *reversal* leads to a balanced network.
  - Count the smallest number of lines who *removal* leads to a balanced network.
  - Fortunately, these are the same.

Line Index of Imbalance

- Reminder: a balanced network can be partitioned into two or more clusters (called plus-sets) where:
  - All the positive arcs are within plus-sets.
  - All the negative arcs are between plus-sets.
- Having established a partition as close to balance as possible:
  - count the number of positive lines *between* plus-sets, P.
  - count the number of negative lines *within* plus-sets, N.
  - Imbalance = $\alpha N + (1 - \alpha)P$.
  - Where $0 \leq \alpha \leq 1$.
- This provides a nice measure of imbalance for a signed network.

Thinking about Three (Linked) Processes

- Processes
  - Reciprocity: $i \rightarrow j \Leftrightarrow j \rightarrow i$.
  - Transitivity: $i \rightarrow j$ and $j \rightarrow k \Rightarrow i \rightarrow k$.
  - Balance and imbalance
- Measures
  - Proportion of reciprocated ties.
  - Proportion of transitive triples.
  - Amount of imbalance.

Three (Linked) Processes: Results

Doreian *et al.* (1997) looked at these measures over time and showed (using the Newcomb data):

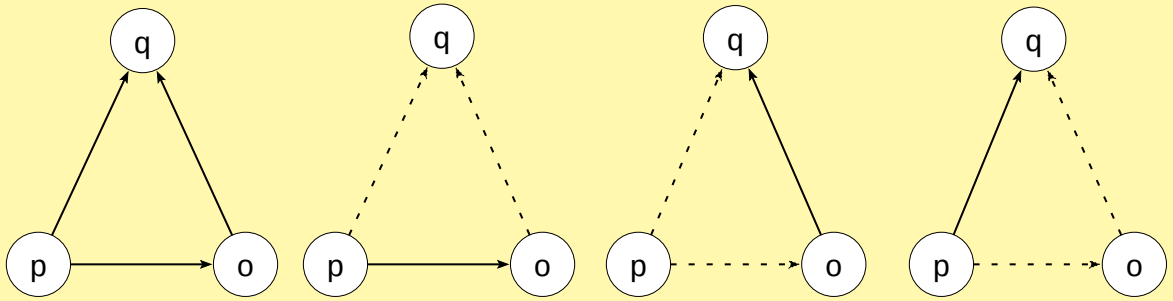
- Results
  - Reciprocity reached its peak level the fastest (in three weeks).
  - Transitivity reached its peak next in time (about seven weeks).
  - Structural balance was the slowest to reach its minimum (non-zero) value (after thirteen weeks).
- And raised the following questions, given the *different time scales*:
  - Is there a theoretical statement of how these processes work together?
  - Can we build a dynamic model of the coupled processes?

Another Look at Balance over Time

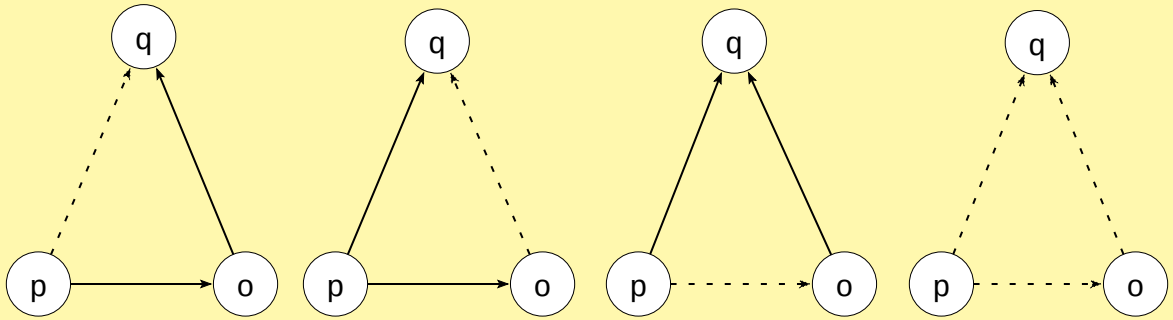
- Consider triples and how they complete.
- Doreian and Krackhardt (2001) looked at $p \rightarrow o$ (+ or -) and $o \rightarrow q$ (+ or -) at one time point (pre- transitive balance conditions) and asked how triples were completed at a later time point with $p \rightarrow q$.
- Their expectation, given that we know the amount of (overall) imbalance declined, was:
  - Balanced triple increase in frequency.
  - Imbalanced triples decrease in frequency.

Signed *poq* triples

Four Balanced Triples



Four Imbalanced Triples



Counting Cycle Completions

Possible triples:

Balanced			Imbalanced		
$p \rightarrow o$	$o \rightarrow q$	$p \rightarrow q$	$p \rightarrow o$	$o \rightarrow q$	$p \rightarrow q$
P	P	P	P	P	N
P	N	N	P	N	P
N	P	N	N	P	P
N	N	P	N	N	N

Counting Cycle Completions: Results

Triple	Tie 1 $p \rightarrow o$	Tie2 $o \rightarrow q$	Tie 3 $p \rightarrow q$	Balanced	Change	Supports Balance
PPP	+	+	+	Yes	Increased	Yes
PPN	+	+	-	No	Decreased	Yes
PNP	+	-	+	No	Decreased	Yes
PNN	+	-	-	Yes	Increased	Yes
NPP	-	+	+	No	Increased	No
NPN	-	+	-	Yes	Decreased	No
NNP	-	-	+	Yes	Decreased	No
NNN	-	-	-	No	Increased	(Yes)

Counting Cycle Completions: Implications

The empirical world is more complex than simple structural balance suggests.

- Overall, there was movement towards balance.
- 'The structural balance process' is a set of processes.
- BUT, only some of the changes are consistent with balance.
- Balance processes work only if $p \rightarrow o$ is a positive tie.
- And works when the $p \rightarrow o$ is negative only for the all negative triple.
- We should also allow for the presence of other processes.

Rethinking the Cartwright and Harary Generalization

On the positive side.

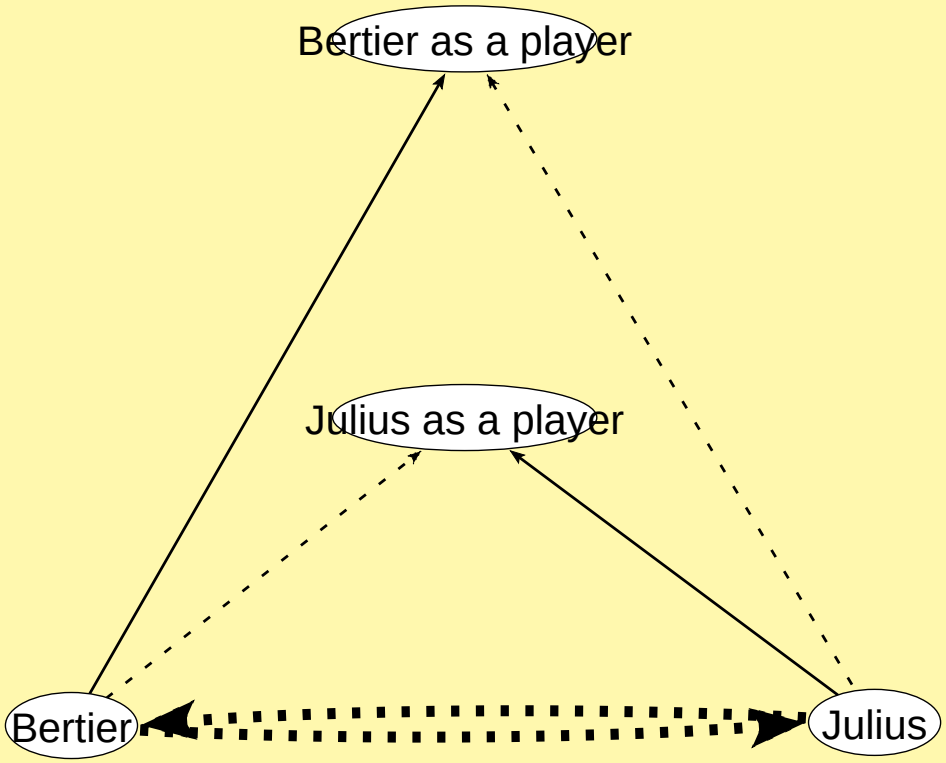
- It was a neat use of mathematics to generalize an intuitive formulation.
- As a result, progress was made in our ability for decribe the overall (partition) structure of signed social network.

On the negative side

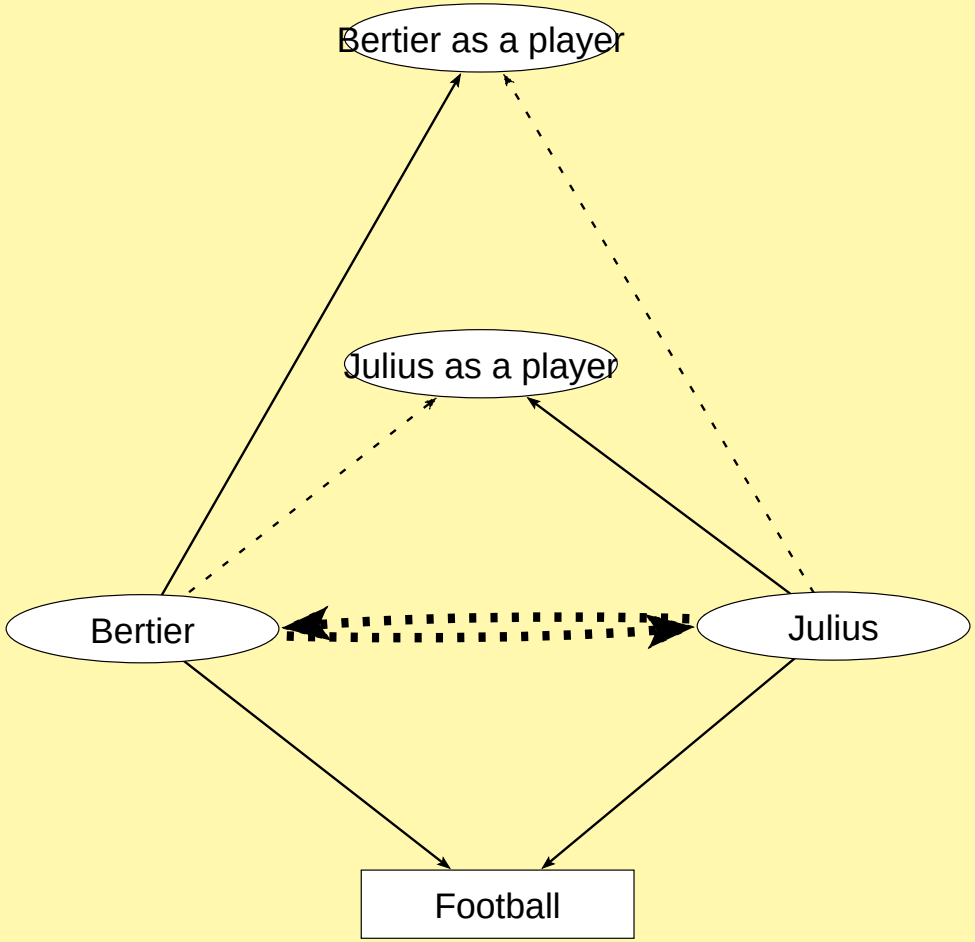
- But we lost some things:
- Gone was happened inside people's heads (and was core to Heider's ideas).
- Gone also were the unit formation relations.

Remember the Titans: Bertier and Julius

Bertier, Julius and Views of Each Other



Bertier, Julius and Football



Summary

- Structural balance seems an intersting idea with a good formalization.
- Empirical evidence has been mixed but the few temporal signed network data sets suggest overall movement towards balance.
- The empirical world is more complicated than the simple version of the theory suggests.
- We need to expand the balance theoretic framework to incorporate one or more of::
  - The idea of considering structural balance as a set of processes.
  - Models with additional processes in play.
  - The inclusion of unit formation relations.
  - The inclusion of what is going on in the minds of actors.